

**D. Jackson:**

**Technique for Recovering Images from Confined Spaces**

**TECHNIQUE FOR RECOVERING IMAGES FROM TIGHTLY CONFINED  
SPACES**

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**ABSTRACT**

This work describes an approach for recovering images from tightly confined spaces using multimode and analytically demonstrates that the concept is sound. The proof of concept draws upon earlier works that concentrated on image recovery after two-way transmission through a multimode fiber and also sending images one-way through a multimode fiber; both used four wave mixing techniques. This approach also uses four wave mixing, but utilizes two fibers to capture the image at some intermediate point (accessible by the fibers, but which is visually inaccessible).

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### **I. INTRODUCTION**

All-optical two dimensional image transmission from point A<sub>0</sub> to point B<sub>0</sub>, over a single fiber had long been a goal in communications, optical interconnection, and medical applications until Volyar et.al.<sup>1</sup> demonstrated one way image transmission through a multimode fiber in 1991. However, their approach requires a bulky optical configuration at point A<sub>0</sub>, which makes it difficult to apply this technique to the observation of phenomena in close or tight spaces. In fact, the use of a less bulky setup in the vicinity of point A<sub>0</sub> is important to a number of remote imaging applications. For example to facilitate close inspections in small spaces, robotic optical imaging sensors for planetary surface exploration, real-time *in vivo* diagnostics in surgical applications, long term physiological and anatomical observations *in vivo* as well as long term undetected surveillance activities. Long term embedding of such *in vivo* monitors (i.e. over several months to years) have applications potential in the study of microgravity effects on biological systems, and providing day to day feedback on the progress of healing during treatment in clinical medical situations.

A compact method for recovering images from tightly confined spaces using a pair of multimode fibers is illustrated in Figure 1 above. The Fabry Perot interferometer provides a means for switching between a calibration mirror and the image that one wishes to retrieve. Below the Fabry Perot interferometer, a SELFOC lens is inserted as a spacer that maximizes the overlap of the modes scattered between the two fibers. To recover the image, the following steps are performed:

- (1) Calibration of the fiber is performed with a blank reflecting surface at the image location, A<sub>0</sub>. During calibration, the phase distortion information is concurrently recorded using a hologram at C<sub>0</sub>.
- (2) The blank reflecting surface is replaced by the image sample, and irradiated with the predistorted conjugate beam which is read out from the hologram. Image information is concurrently sampled at point B<sub>0</sub>.

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Unlike the earlier image recovery analyses which assume a steady state<sup>2</sup>, this treatment assumes that the image recovery at point B<sub>0</sub> occurs in a time,  $\tau_0 \ll \tau_{erase}$ , where  $\tau_0$  is the sampling time of the readout CCD, and  $\tau_{erase}$  is the erasure time, of the crystal while being illuminated by the read beam. On the basis of this assumption, the rest of this paper is organized in the following manner. First, the classical derivation of two-way image transmission through a fiber is reviewed. Then, the calibration setup using two fibers as shown in Figure 1 and summarized in steps (1) and (2) above is derived (i.e. when the Fabry Perot is tuned to its high reflectivity state). Finally, the interferometer is tuned to its maximum transmission mode and an analysis is performed to determine the nature of the image function at the B<sub>0</sub> output position.

## II. TWO-WAY IMAGE TRANSMISSION THROUGH A FIBER

Historically, images have been transmitted through multi-mode fibers, only for the class of situations where the image is launched from point A<sub>1</sub> into the fiber and is returned to point A<sub>1</sub> via the same fiber (see Figure 2), after a phase conjugating step at point B<sub>1</sub>. Inputting the image,  $f(x, y, z = 0, t)$ , at point A<sub>1</sub> and propagating it through the fiber to point B<sub>1</sub> distorts the image information beyond recognition because of modal dispersion<sup>2-7</sup>. Before Volyar et. al.'s demonstration, the only way to recover the image was to perform the conjugating step shown in Figure 2 to unravel the phase. To recover images from a tightly confined space, the same approach is used to unravel the phase, but the single fiber is replaced by a pair of fibers of lengths L<sub>1</sub> and L<sub>2</sub>, spliced together with a Fabry Perot etalon.

This can be shown analytically as follows:

At the input to the fiber, one has an optical field

$$f_1(x, y, z = 0, t) = \sum_{m=0}^N \sum_{n=0}^N A_{mn} E_{mn}(x, y) \exp[i\omega t] \quad [1]$$

where  $E_{mn}(x, y)$  is the m,nth spatial mode function of the particular waveguide used and  $A_{mn}$  is the amplitude coefficients for the m,nth mode. Generally after the image field is translated a distance L down the fiber, the image is scrambled due to the different phase factors,  $\beta_{mn}$ , for each of the modes. Thus the field at the output of the fiber has the form:

$$f_2(x, y, L, t) = \sum_{m=0}^N \sum_{n=0}^N A_{mn} E_{mn}(x, y) \exp[i(\omega t - \beta_{mn} L)] \quad [2]$$

which can be rewritten as

$$f_2(x, y, L, t) = \sum_{m=0}^N \sum_{n=0}^N C_{mn} E_{mn}(x, y) \exp[i\omega t] \text{ where } C_{mn} = A_{mn} \exp[-i\beta_{mn} L]$$

which seriously distorts the image information.

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The image information can be recovered or unscrambled, if the complex conjugate of the spatial portion of the input field can be generated and launched into a section of fiber that is exactly identical in length and all other characteristics to the first fiber. (In demonstrations, the same fiber is usually used to insure that all of the characteristics are faithfully reproduced.) To obtain the phase conjugation of a given field, one takes the complex conjugate of the spatial part of the field only and leaves the time dependent component untouched. By introducing a phase conjugating medium at the output of the fiber, one can generate, via either 2-wave mixing or 4-wave mixing, a returning phase conjugated field:

$$\begin{aligned} f_3(x, y, L, t) &= \sum_{m=0}^N \sum_{n=0}^N C_{mn}^* E_{mn}^*(x, y) \exp[i\omega t] \\ &= \sum_{m=0}^N \sum_{n=0}^N A_{mn}^* E_{mn}^*(x, y) \exp[i(\omega t + \beta_{mn} L)] \end{aligned} \quad [3]$$

This conjugate field has just exactly the right initial phase to allow it to backward propagate through the fiber and unravel any phase scrambling done by the original trip. Thus after reverse transmission through the fiber, the output field has the form

$$\begin{aligned} f_4(x, y, 0, t) &= \sum_{m=0}^N \sum_{n=0}^N A_{mn}^* E_{mn}^*(x, y) \exp[i(\omega t + \beta_{mn} L - \beta_{mn} L)] \\ &= \sum_{m=0}^N \sum_{n=0}^N A_{mn}^* E_{mn}^*(x, y) \exp[i\omega t] \end{aligned} \quad [4]$$

which restores the original image.

### III. ONE-WAY IMAGE TRANSMISSION THROUGH A FIBER

Consider the situation where one wants to illuminate a sample with a remote light source using a single fiber of length  $L_2$  as a conduit as shown in Figure 1 and then retrieve image information with the light collected by a second fiber of length  $L_1$ .

#### A. Calibrating the fiber characteristics

In order to recover the image, one must first calibrate the characteristics of the fiber by writing a hologram in a recording medium after the input illumination,  $f_1(x, y, z = 0, t)$ , is injected at point  $B_0$ , then collected in fiber  $L_2$  after scattering off of a reflector at point  $A_0$  (see Figure 3 below), and is finally output at point  $C_0$  as  $f_2(x, y, z = L_2, t)$  with whatever phase scrambling this optical path through the system accrues.

Thus,

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$$f_1(x, y, z = 0, t) = \sum_{i=0}^N \sum_{j=0}^N a_{ij} F_{ij}(x, y) \exp[i\omega t] \quad [5]$$

$$f_1(x, y, z = L1, t) = \sum_{i=0}^N \sum_{j=0}^N a_{ij} F_{ij}(x, y) \exp[i(\omega t - \beta_{ij} L1)] \quad [6]$$

And the image injected into the L2 fiber is expressed as a linear superposition of the eigenmodes of the L1 fiber.

$$f_2(x, y, z = 0, t) = \sum_{k=0}^N \sum_{l=0}^N b_{kl} F_{kl}(x, y) \exp[i\omega t] \quad [7]$$

$$f_2(x, y, z = L2, t) = \sum_{k=0}^N \sum_{l=0}^N b_{kl} F_{kl}(x, y) \exp[i(\omega t - \beta_{kl} L2)] \quad [8]$$

The coefficients  $b_{kl}$  can be expressed as linear superpositions of the eigenmode coefficients for the  $a_{ij}$  coefficients:

$$b_{kl} = \sum_{i=0}^N \sum_{j=0}^N h_{kl ij} a_{ij} \exp[-i(\beta_{ij} L1)] \quad [9]$$

The hologram is written by interacting the fiber output beam,

$$f_2(x, y, z = L2, t) = \sum_{k=0}^N \sum_{l=0}^N b_{kl} F_{kl}(x, y) \exp[i(\omega t - \beta_{kl} L2)], \text{ and a reference beam}$$

$f_r(x, y, z, t) = \sum_{k=0}^N \sum_{l=0}^N r_{kl} F_{kl}(x, y) \exp[i\omega t]$  in a holographic recording medium such as BaTiO<sub>3</sub>, where  $r_{kl}$  is the amplitude coefficient of each mode of the reference beam.

$$f_{holo}(x, y, z, t) = f_r(x, y, z, t) + f_2(x, y, z, t) \quad [10]$$

$$\begin{aligned} & |f_{holo}(x, y, z)|^2 \\ &= \sum_{k=0}^N \sum_{l=0}^N \sum_{k'=0}^N \sum_{l'=0}^N \{ r_{kl} r_{k'l}^* + r_{kl} b_{k'l}^* \exp[i\beta_{k'l} L2] + b_{kl} r_{k'l}^* \exp[-i\beta_{k'l} L2] + b_{kl} b_{k'l}^* \} F_{kl}(x, y) F_{k'l}^*(x, y) \end{aligned} \quad [11]$$

It is the second and third terms that carry the phase information needed to reconstruct a hologram. To read out the original image input to the holographic medium, one simply injects the reference beam. To read out the conjugated image, one must input the conjugate of the reference beam. Assuming, for example, that the original reference is a plane wave, the conjugate is a plane wave incident from the opposite direction of the reference beam as shown in Figure 4. This produces a conjugated output:

$$f_3(x, y, z = L2, t) = \sum_{p=0}^N \sum_{q=0}^N \sum_{k=0}^N \sum_{l=0}^N \sum_{k'=0}^N \sum_{l'=0}^N [r_{kl} b_{k'l}^* \exp[i\beta_{k'l} L2] r_{pq}^*] F_{kl}(x, y) F_{k'l}^*(x, y) F_{pq}^*(x, y) \exp[i\omega t] \quad [12]$$

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Since both  $f_r(x, y, z, t)$  and  $f_2(x, y, z, t)$  are expressed as linear superpositions of the normal modes of the fiber, their products are zero except when  $k=k'$  and  $l=l'$ . Hence:

$$f_3(x, y, z = L2, t) = \sum_{k=0}^N \sum_{l=0}^N b_{kl}^* F_{kl}^*(x, y) \exp[i(\omega t + \beta_{kl} L2)] \quad [13]$$

$$f_3(x, y, z = 0, t) = \sum_{k=0}^N \sum_{l=0}^N b_{kl}^* F_{kl}^*(x, y) \exp[i\omega t] \quad [14]$$

where  $b_{kl}^* = b_{kl}^* |f_r(x, y)|^2 = b_{kl}^* \sum_{p=0}^N \sum_{q=0}^N \sum_{k=0}^N \sum_{l=0}^N [r_{kl} r_{pq}^*] F_{kl}(x, y) F_{pq}^*(x, y)$

Again the reflected image collected by fiber L1 can be expressed as a linear superposition of the eigenmodes of the L2 fiber. Hence:

$$d_{mn} = \sum_{k=0}^N \sum_{l=0}^N g_{mnkl} b_{kl}^* |f_r(x, y)|^2 \quad [15]$$

$$\text{where } b_{kl}^* = \sum_{i=0}^N \sum_{j=0}^N h_{kl ij} a_{ij}^* \exp[i(\beta_{ij} L1)],$$

and

$$d_{mn} = \sum_{k=0}^N \sum_{l=0}^N \sum_{i=0}^N \sum_{j=0}^N g_{mnkl} h_{kl ij}^* a_{ij}^* |f_r(x, y)|^2 \exp[i(\beta_{ij} L1)] \quad [16]$$

As long as  $s \ll D^2 / \lambda$  where  $s$  is the nominal distance from the fiber to the reflecting surface,  $D$  is the aperture of the fiber, and  $\lambda$  is the optical wavelength, there is enough phase information collected to completely reconstruct the original path through both fibers. Thus one requires that  $g_{mnkl} = h_{kl ij}^{-1} = \frac{H_{klmn}^{(mn)}}{\det H^{(mn)}}$  where  $H \equiv [h_{mnkl}]$  with all real coefficients and  $H_{klmn}^{(mn)}$  is the cofactor of  $h_{mnkl}$  along the  $m$ th and  $n$ th columns. Finally

$$\sum_{i=0}^N \sum_{j=0}^N \sum_{k=0}^N \sum_{l=0}^N h_{kl ij} H_{klmn}^{(mn)} = \begin{cases} \det H^{(mn)} & \text{if } i = m; j = n \\ 0 & \text{if } i \neq m; j \neq n \end{cases} \quad [17]$$

consequently the summation is restricted to  $i=m$  and  $j=n$ , so that the output field simplifies to the form:

$$f_4(x, y, z = L1, t) = \sum_{m=0}^N \sum_{n=0}^N d_{mn}^* F_{mn}^*(x, y) \exp[i(\omega t + \beta_{mn} L1)] \quad [18]$$

$$f_4(x, y, z = L1, t) = \sum_{m=0}^N \sum_{n=0}^N a_{mn}^* |f_r(x, y)|^2 F_{mn}^*(x, y) \exp[i(\omega t + \beta_{mn} L1)] \quad [19]$$

$$f_4(x, y, z = 0, t) = \sum_{m=0}^N \sum_{n=0}^N a_{mn}^* |f_r(x, y)|^2 F_{mn}^*(x, y) \exp[i\omega t] \quad [20]$$

## B. Recovering the Image Information

Once the fiber characteristics have been calibrated, the image can be recovered. The above calculation assumes that the surface at point A0 is a uniform reflector like a

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mirror. If image information is now imposed at the point of reflection, A<sub>0</sub>, it modifies the amplitude of the  $b_{kl}$  and  $d_{mn}$  coefficients in equation 9 and 16 to include the image information such that:

$$b_{kl} = \sum_{i=0}^N \sum_{j=0}^N v_{kl} h_{kl ij} a_{ij} \exp[-i(\beta_{ij} L1)] = \sum_{i=0}^N \sum_{j=0}^N h_{kl ij}^1 a_{ij} \exp[-i(\beta_{ij} L1)] \quad [21]$$

$$\begin{aligned} d_{mn} &= \sum_{k=0}^N \sum_{l=0}^N \sum_{i=0}^N \sum_{j=0}^N \eta_{mn} g_{mnkl} h_{kl ij}^* a_{ij}^* |f_r(x, y)|^2 \exp[i(\beta_{ij} L1)] \\ &= \sum_{k=0}^N \sum_{l=0}^N \sum_{i=0}^N \sum_{j=0}^N g_{mnkl}^1 h_{kl ij}^* a_{ij}^* |f_r(x, y)|^2 \exp[i(\beta_{ij} L1)] \end{aligned} \quad [22]$$

where  $v_{kl}$  and  $\eta_{mn}$  contain phase and amplitude information pertaining to the image. However, at the moment that the image information is introduced, there is no phase correction for equation 21 written into the crystal. Hence  $f(x, y, z = 0, t)$  is initially unchanged as it illuminates the sample. Plugging equation 22 into equation 18, one sees that equation 20 at the output now becomes:

$$f_4(x, y, z = 0, t) = \sum_{m=0}^N \sum_{n=0}^N \eta_{mn} a_{mn}^* F_{mn}^*(x, y) \exp[i(\omega t)] \quad [23]$$

hence the image information introduced at point A<sub>0</sub> and illuminated by light injected at point C<sub>0</sub>, is now recovered at point B<sub>0</sub>. This solution holds for  $\tau_0 \ll \tau_{erase}$ . Note that as the sampling duration begins to approach  $\tau_{erase}$ , the coefficients adjust such that the transfer matrices compensate each other and  $g_{mnkl}^1 = (h_{kl ij}^1)^{-1}$ . When this condition is met, the original image is returned as in equation 20.

#### IV. CONCLUSIONS

This paper has analytically shown a technique for recovering images from tightly confined spaces. This image recovery process entails two key steps. First, the path through the fiber or any other image scrambling medium must be calibrated by imposing a calibration reflector at the position where the image is to be detected. Then, after a hologram of the accumulated phase scrambling is written into the recording medium, the calibration reflector is replaced by the sample image and that information is output unscrambled at point B<sub>0</sub>. The two technologies illustrated in Figure 1, which are key to recovering this image information are:

(i) a photorefractive crystal such as BaTiO<sub>3</sub> which exhibits a time lag of 1 to 20 seconds in the erasure of the calibration hologram during the image readout step.

(ii) a MEMS Fabry Perot micro-interferometer<sup>8</sup>, which is both easy to mount at the end of a fiber, and can be tuned between the two optical throughput states of 100%

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reflection and 100% transmission. These MEMS devices have the additional advantage that they can be mass produced at low cost.



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## **FIGURE CAPTIONS**

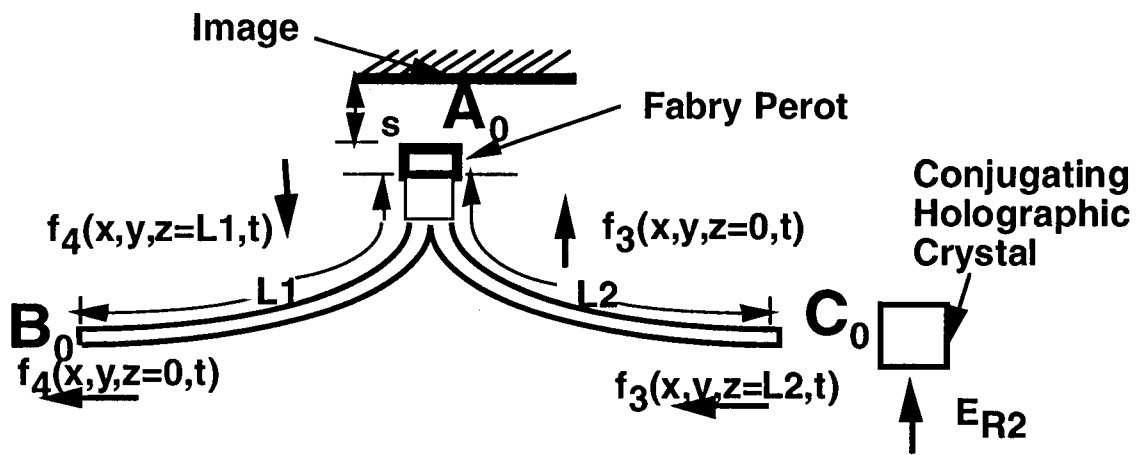
**Figure 1** Setup for Remote Image Sampling with Fibers

**Figure 2** Schematic of the use of phase conjugation to recover an image after transmission through a multimode fiber.

**Figure 3.** Setup for calibrating fiber distortion by writing hologram.

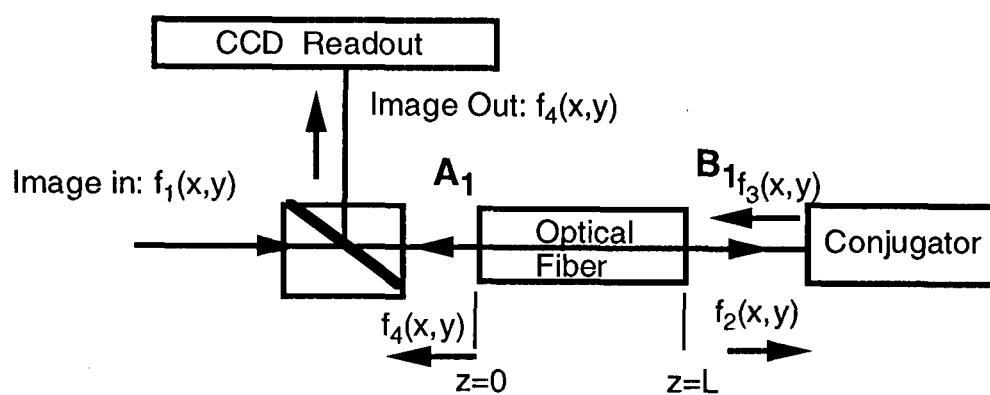
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